Chapter 3: Motion and Forces

Goals of Period 3

Section 3.1: To define speed and velocity as rates
Section 3.2: To explain the cause of changing velocity – forces
Section 3.3: To define changes in velocity - acceleration
Section 3.4: To examine the acceleration of falling objects
Section 3.5: To calculate the force required to accelerate an object

Our Period 2 discussion of energy forms and conversions illustrated some of the many ways in which energy is used. Forces are responsible for energy. Period 3 introduces the concept of forces acting on objects to make them move. Since motion can involve either a fixed velocity or a change in velocity (acceleration), our explanation of forces includes a discussion of speed, velocity, and acceleration.

3.1 More about Rates: Speed and Velocity

As explained in Period 1, ***per*** means *for each* and designates a ratio. For example, 55 miles per hour (55 MPH) can be written as the fraction 55 miles/hour, which tells us how many miles (55) are traveled in one hour. This ratio is an example of a ***rate***, a ratio that tells how fast or how often something occurs. For example, 25 miles per gallon (25 MPG) is a rate that tells how many miles we can travel on one gallon of gas.

We begin the discussion of rates by carefully defining the quantities used. In physics words are used in very specific ways. Such words include distance, time, speed, velocity, and acceleration.

Distance: We use the symbol ***D*** to represent the distance between two points. Distances are measured by comparing the length of the path between two points to standard lengths such as meters or feet. We assume that the length of a meter stick held in our hands is constant, so that a meter stick is one meter long no matter where we are in the Universe.

Time: We may wish to know how much time has passed between events. To measure time, we need an event that repeats, such as the swing of a pendulum clock, where each swing marks a unit of time. Some clocks are more accurate than others. Atomic clocks, which use cesium atoms' radiation to mark units of time, are accurate to billionths of a second. We often use seconds as our standard unit of time. The symbol ***t*** represents the time elapsed between two events.

Speed: Speed is a rate that measures the distance an object travels in a given time and can be written as:

$$\text{Speed} = \frac{\text{Distance traveled}}{\text{Time elapsed}}$$

Using symbols to represent the quantities,

$$s = \frac{D}{t} \qquad \textbf{(Equation 3.1)}$$

where s = speed (in meters/second or miles/hour)
D = distance traveled (in meters or miles)
t = time elapsed (in seconds or hours)

(Example 3.1)

If you drive for 2 hours at a constant speed and travel 120 miles, what is your speed during the trip?

$$s = \frac{D}{t} = \frac{120 \text{ miles}}{2 \text{ hours}} = \frac{60 \text{ miles}}{1 \text{ hour}} = 60 \text{ MPH}$$

Example 3.1 involved motion at a constant speed. In reality, few objects travel at a constant speed for long. In most real life examples, we use the **average** speed of an object as it travels between two points, as shown in example 3.2.

(Example 3.2)

If you drive for 3 hours at an average speed of 55 miles per hour, how far have you traveled?

Equation 3.1 also holds when s is the average speed. We solve Equation 3.1 for D by multiplying both sides by t.

$$s\ t = \frac{D\ \cancel{t}}{\cancel{t}} = D = \frac{55 \text{ miles}}{\cancel{\text{hour}}} \times 3\ \cancel{\text{hours}} = 165 \text{ miles}$$

Speed indicates how fast an object moves, but not the direction of its motion. ***Velocity*** is a rate that specifies the speed *AND* the direction of motion. For example, we can measure speed, such as 40 MPH, with a speedometer, but to determine velocity, we need the direction of motion as well, such as 40 MPH due north.

Concept Check 3.1

a) It takes 5 hours to travel from Columbus to Cleveland and back to Columbus, a total distance of 300 miles. What is your average speed for the trip?

____

b) How long does it take to ride a bicycle 5 miles at an average speed of 10 MPH?

____

Skills and Strategies #4: Using Algebraic Operations

We often must rearrange an equation to solve for a variable. In Example 3.2, we rearranged the equation $s = D/t$ to solve for D. When using such algebraic operations, we follow one important rule:

Always perform the same operations on each side of the equation

Solving for D means putting D by itself on one side of the equation (either the right or the left side). In the case of the equation $s = D/t$, we must cancel t from the right side of the equation and move t to the left side. We need an operation that allows us to cancel t. Multiplying both sides of the equation by t allows cancellation of t from the right side.

$$s\,t = \frac{D\,\cancel{t}}{\cancel{t}} = D \qquad \text{or} \qquad D = s\,t$$

To solve the equation $s = D/t$ for t we begin as before by multiplying both sides of the equation by t.

$$s\,t = \frac{D\,\cancel{t}}{\cancel{t}} = D$$

Next eliminate s from the left side of the equation by dividing both sides by s:

$$\frac{\cancel{s}\,t}{\cancel{s}} = \frac{D}{s} \qquad \text{or} \qquad t = \frac{D}{s}$$

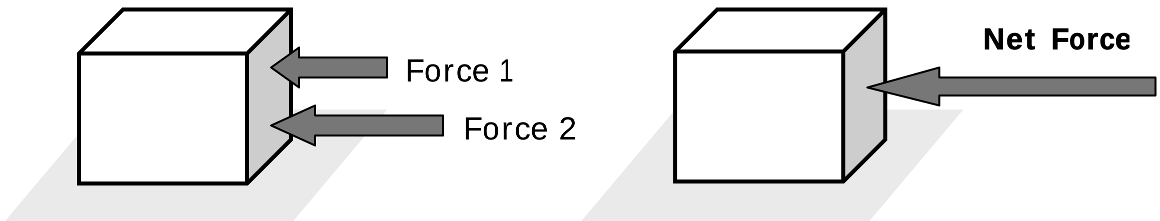
3.2 Introduction to Forces

The velocity of moving objects changes when their speed or direction changes. A ***force*** is required to change the speed or direction of an object. A force is any push or a pull on an object. This definition of force is known as Newton's first law of motion. We discuss Newton's Laws in more detail in Period 5.

To specify a force, we must know the magnitude of the force and its direction. Forces are measured in units of pounds in the English system or in newtons in the metric system. An average sized apple exerts a force on a scale equal to about one newton, or about ¼ of a pound. In class we will measure forces with spring scales, which operate on the same principle as the bathroom spring scale that measures our weight. The greater the force on the scale, the farther the spring in the scale compresses or stretches. If we pull twice as hard on a spring scale, the spring stretches twice as far.

More than one force can act on an object. The sum of all the forces acting is called the ***net force.*** Figures 3.1 and 3.2 show the net force acting on a box, which is sitting on a horizontal surface such as a level floor. Forces acting on an object in the same direction add together. For example, it is easier to move a stalled car with two people pushing in the same direction because their forces add.

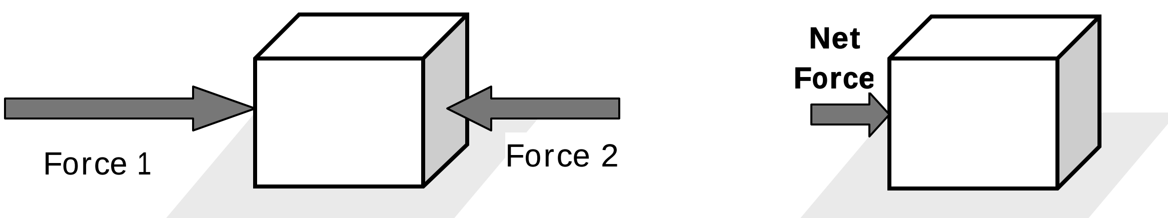
Fig. 3.1 Two Forces Acting on a Box in the Same Direction



Net Force = Force 1 + Force 2

When forces act on an object in opposite directions, the net force is the difference between the forces. Equal forces acting in opposite directions cancel each other. For example, if you hold a box in your hands, the force of gravity pushing down on the box is balanced by the force of your hands pushing up on the box. Since the box does not move, the net force acting on it is zero.

Fig. 3.2 Forces Acting in Opposite Directions



Net force = Force 1 - Force 2

3.3 Changing Velocity: Acceleration

Acceleration is the rate at which velocity changes. Since velocity specifies both a speed and a direction, acceleration is a change in speed (speeding up or slowing down), a change in direction, or change in both speed and direction. When the acceleration is constant, it is expressed by the relationship

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time elapsed}}$$

In terms of symbols, the relationship is

$$a = \frac{v_{final} - v_{initial}}{t} = \frac{v_f - v_i}{t} \qquad \textbf{(Equation 3.2)}$$

where

a = acceleration, in (meters / second) per second = $\frac{\text{meters}}{\text{second}^2}$ or in (miles / hour) per second = $\frac{\text{miles / hour}}{\text{second}}$

v_f = final velocity, in meters/second or miles/hour

v_i = initial velocity, in meters/second or miles/hour

t = time elapsed, in seconds

(Example 3.3)

You are driving at a constant speed of 40 miles per hour. To pass another car, you step on the gas, increasing your speed to 60 miles per hour in 5 seconds. Assuming that your acceleration during these 5 seconds is constant, what is your constant acceleration?

$$a = \frac{v_f - v_i}{t} = \frac{60 \text{ mi/hr} - 40 \text{ mi/hr}}{5 \text{ sec}} = \frac{20 \text{ mi/hr}}{5 \text{ sec}} = \frac{4 \text{ mi/hr}}{\text{sec}}$$

Concept Check 3.2

a) If you drive in a circle at a constant speed of 25 MPH, are you accelerating? Why or why not? ____

b) If you drive along a straight road at a constant speed of 25 MPH, are you accelerating? Why or why not? ______________________________

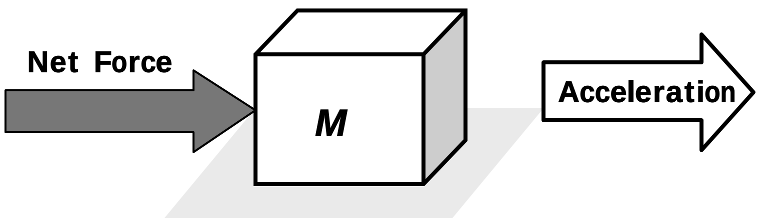
c) If you go from a complete stop to traveling at 30 miles/hour in 6 seconds, what is your acceleration? _____________

d) If you continue to accelerate at the same rate as in part c), what will be your speed 7 seconds after you began driving? _____________

3.4 The Force Required to Accelerate an Object

The amount of acceleration of an object produced by a net force on the object depends on the magnitude (amount) of the net force and the mass of the object. Mass is a measure of the amount of matter an object contains. We will usually measure mass in units of grams or kilograms. The net force acting on the object is equal to the product of an object's mass times the acceleration resulting from the force. As shown in Figure 3.3, the net force and the acceleration must be in the same direction.

Fig. 3.3 A Net Force Accelerates a Box



Net Force = mass x acceleration

or

$$F = Ma \qquad \textbf{(Equation 3.3)}$$

where F = net force (in newtons)
M = mass (in kilograms)
a = acceleration (in meters/second^2)

Force is measured in units of newtons (N). A newton is a combination of the units of mass times acceleration (kg x m/s^2). For example, one newton could equal one kilogram of mass accelerating at the rate of one meter per second^2. When using force in units of newtons and mass in kilograms, acceleration must be expressed in meters/second^2. This relation between force, mass, and acceleration, which is given by Equation 3.3, is Newton's second law of motion.

(Example 3.5)

How much force is required to accelerate a 4000 kg truck at a constant rate of 5 meters/second^2? (Ignore any frictional forces.)

$$F = Ma = 4000 \text{ kg} \times 5 \text{ m/s}^2 = 20{,}000 \text{ kg m/s}^2 = 2 \times 10^4 \text{ newtons}$$

Example 3.5 of Newton's Law uses metric units: force in newtons, mass in kilograms, and acceleration in meters/second^2. Equations can be solved using either metric or English units as long as the units used are consistent in each equation. Tables 3.1 and 3.2 provide metric and English unit equivalents useful for solving problems.

Table 3.1: Metric and English Units and Their Symbols

Quantity	Symbol	Metric Units	Metric Unit Abbreviation	English Unit	English Unit Abbreviation
Mass	***M***	kilograms	kg	slugs*	slug*
Time	***t***	seconds	s	seconds	s
Distance	***D***	meters	m	feet	ft
Velocity	***v***	meters/second	m/s	feet/second	ft/s
Acceleration	***a***	meters/second^2	m/s^2	feet/second^2	ft/s^2
Force	***F***	newtons	N = kg m/s^2	pounds	lb
Energy	***E***	joules	J = kg m^2/s^2	foot-pounds	ft-lb

**The English unit for mass, the slug, is seldom used.*

Table 3.2: Metric and English Equivalents

1 mile	=	5,280 feet		
1 mile	=	1,609 meters	=	1.609 kilometers
1 foot	=	0.305 meters		
1 pound	=	4.45 newtons		
1 hour	=	3,600 seconds		

Skills and Strategies #5: Putting Units to Work for You

Including units with each quantity gives a shortcut to problem solving and an easy way to check answers. For example, if you mistakenly solved Newton's force equation, $\boldsymbol{F} = \boldsymbol{M}\ \boldsymbol{a}$ for the mass, ***M***, and got

$$\boldsymbol{M} = \boldsymbol{F}\ \boldsymbol{a} \quad \longleftarrow \textbf{NOT Correct}$$

you could find the mistake by checking the units of the answer. After canceling, a correct solution has the same units on both sides of the equation. Checking the units of the correct solution to the equation, we find

$$\boldsymbol{M} = \frac{\boldsymbol{F}}{\boldsymbol{a}}$$

$$\text{kg} = \frac{\text{kg}\ \cancel{\text{m}/\text{s}^2}}{\cancel{\text{m}/\text{s}^2}} = \text{kg}$$

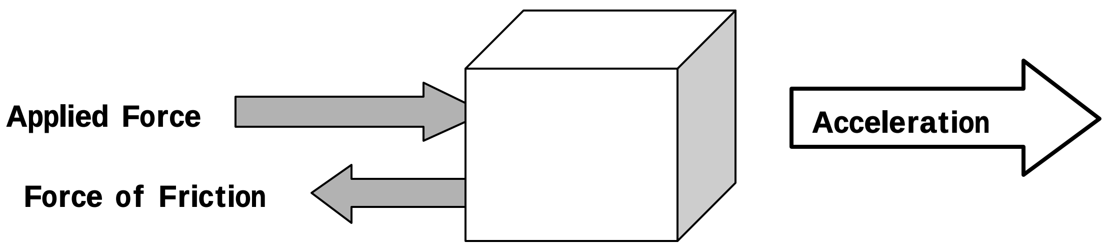
We have used units of mass in kilograms (kg), force in newtons (N or kg m/s^2), and acceleration in meters per second^2 (m/s^2). After canceling the units common to the numerator and the denominator, we end with kilograms on both sides of the equation.

You can save time in the long run by including units with each quantity and in each step of problem solutions!

Newton's Law, ***F = M a***, tells us that the amount of acceleration of an object is proportional to the net force acting on it. From the equation, we might expect that giving an object a push with a force ***F*** would cause it to accelerate forever, since Newton's Law does not specify the duration of the acceleration. However, our everyday experience tells us that a giving an object a push causes it to slide and then come to a stop. How can we explain our experience in terms of Newton's Law?

Newton's Law requires that we consider **all** of the forces acting on an object. In equation 3.3, ***F*** is the **net** force acting. A net force is the sum of the forces acting in the same direction or the difference of forces acting in opposite directions. In the case of a sliding box, the force of friction acts in the direction opposite to the motion of the box and slows its acceleration. Figure 3.4 illustrates an applied force greater than the force of friction. The net force on the box is the difference between the applied force and the force of friction. In this case, the net force accelerates the box in the direction of the applied force.

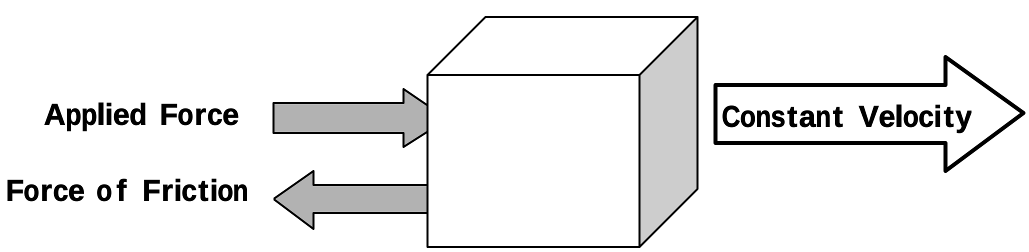
Fig 3.4 An Applied Force Larger than the Force of Friction



If the applied force is greater than the force of friction, the box accelerates across a level floor in the direction of the applied force.

Next we consider an applied force equal to or less than the force of friction. If an object is at rest, an applied force greater than the force of friction is necessary to start the object in motion. Therefore, an object at rest remains at rest when a force equal to or less than the force of friction is applied. To an object in motion we can continuously apply an amount of force on the object equal to the force of friction so that the object moves at a constant velocity. In the case of constant velocity, the applied force equals the force of friction between the object and the surface it is sliding across. Figure 3.5 illustrates a box moving at a constant velocity. Since the applied force is equal to and in the opposite direction of the force of friction, the net force on the box is zero.

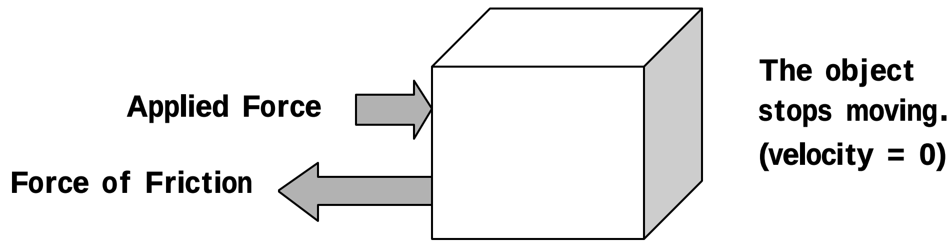
Fig 3.5 An Applied Force Equal to the Force of Friction



If the applied force is equal to the force of friction, the box moves with constant velocity across a level floor in the direction of the applied force.

If a force less than the force of friction is applied to a moving object, the object eventually come to rest. Figure 3.6 illustrates this situation.

Fig 3.6 An Applied Force Less than the Force of Friction



If the box is in motion and the applied force is less than the force of friction, the box comes to a stop.

Concept Check 3.3

a) You apply a horizontal force to the side of a box to push it across a level floor. What must be the net force on a 5 kilogram box for it to move with an acceleration of 2 meters/second each second (2 meters/second^2)?

____

b) Once this box is moving, you must apply a force of 6 N to keep it moving at a constant velocity in a straight line. How great is the force of friction between the bottom of the box and the surface it slides across? __________

When an object accelerates in air, water, or other medium, it must push the molecules of the medium out of its way. As the object's velocity increases, it must push aside increasing amounts of the medium per unit time. The greater the amount of the medium it must push aside, the greater the resistance to its motion. This resistance is a frictional force between the surface of the object and the medium it moves through. The amount of the frictional force increases as the velocity of the object increases. Eventually the object reaches a velocity at which the frictional force is equal to the applied force. The two equal forces acting in opposite directions cancel, and the object moves at constant velocity.

3.5 The Acceleration of Falling Objects

So far, we have considered a net force accelerating an object in a horizontal direction. Next, we discuss the acceleration caused by the force of gravity. The force of gravity causes unsupported objects near the Earth to accelerate toward the Earth's center. We will learn more about how the gravitational force operates in Periods 4 and 5. In this period, we examine the acceleration of falling objects.

As objects fall, the force of gravity accelerates them and their velocity increases. Near the Earth, the force of gravity accelerates a falling object by 9.8 meters per second each second, which is written as 9.8 m/s^2 in metric units or 32 ft/s^2 in English units. The acceleration of gravity is designated by the letter ***g***. If we ignore the resistance of the air pushing against a falling object, which slows its fall, we can calculate the object's velocity after it has fallen for a given length of time.

(Example 3.6)

You drop a penny from the top of a tall building. If you ignore the slowing effect of air resistance, what is the penny's velocity after it has fallen for 3 seconds?

The penny's initial velocity is zero. Once you release the penny, it accelerates at the rate of ***g*** = 9.8 m/s^2. After one second has passed, the penny has accelerated from a velocity of 0 m/s to 9.8 m/s. After another second has passed, the penny has increased its velocity by 9.8 m/s and falls with a velocity of 19.6 m/s. After the third second has passed, the penny has increased its velocity by another 9.8 m/s to 29.4 m/s as shown in the table.

Time Elapsed	Velocity
1 sec	9.8 m/s^2 x 1 sec = 9.8 m/s
2 sec	9.8 m/s + (9.8 m/s^2 x 1 sec) = 19.6 m/s.
3 sec	9.8 m/s + 9.8 m/s + (9.8 m/s^2 x 1 sec) = 29.4 m/s.

Example 3.6 illustrates an important concept: when acceleration is constant, the increase in velocity during each time period is the same – in this example, 9.8 m/s.

We could also have answered Example 3.6 by solving Equation 3.2, shown below, for the final velocity v_f.

$$a = \frac{v_{final} - v_{initial}}{t} = \frac{v_f - v_i}{t}$$

First, multiply both sides of the equation by ***t***, which allows us to cancel ***t*** from the right side.

$$a\,t = \frac{(v_f - v_i)\,\cancel{t}}{\cancel{t}} \quad or \quad a\,t = v_f - v_i$$

Next, add v_i to both sides of the equation to cancel it from the right side.

$$a\,t + v_i = v_f - \cancel{v_i} + \cancel{v_i}$$

The result is Equation 3.4, another expression for velocity in terms of a constant acceleration.

$$v_f = a\,t + v_i \qquad \textbf{(Equation 3.4)}$$

where

a	=	constant acceleration (in meters/second^2 or miles/hour per second)
v_f	=	final velocity (in meters/second or miles/hour)
v_i	=	initial velocity (in meters/second or miles/hour)
t	=	time elapsed (in seconds)

Using Equation 3.4 to solve Example 3.6, where the initial velocity is zero, gives

$$v_f = a\,t + v_i = (9.8 \text{ m/s}^{\not 2} \times 3 \not{s}) + 0 \text{ m/s} = 29.4 \text{ m/s}$$

Concept Check 3.4

a) A car traveling at 40 miles/hour accelerates to pass a truck. Suppose that the car maintains a constant acceleration of 5 miles/hour per second for 4 seconds. What is the car's final velocity?

____

Suppose that at the same time you dropped the penny from the roof of a tall building, your friend standing beside you drops a bowling ball. If we again ignore air resistance, which object will hit the ground first – the penny or the bowling ball? How does an object's mass affect the rate of its acceleration? As you will see in class, when we ignore air resistance, the mass and shape of the object does not affect its acceleration by the force of gravity.

A vacuum means there is no air or other medium for the object to push aside. In a vacuum, the penny and the bowling ball reach the ground at the same time. This may seem to contradict your experience since the acceleration of an object pulled across a level floor depends on the mass of the object. The more massive the object, the slower its acceleration. However, this is NOT the case for objects falling under the force of gravity. For them, the acceleration is always the same regardless of their mass (9.8 m/s^2 or 32 ft/s^2). We will discuss this further in Chapter 5.

Of course, in the real world air resistance is present to slow the fall of falling objects. We have seen that an object moving horizontally through air or fluid reaches a constant velocity when the force accelerating it equals the force needed to push aside the air or fluid. Similarly, a falling object must push air molecules out of its way as it falls. The object reaches a constant velocity when the force of the friction of air molecules pushing up equals the force of gravity pulling down. The greater the surface area of the falling object, the greater the upward force of air resistance. A parachutist drifts to earth at a safe constant velocity because the large surface area of the parachute creates enough friction as it pushes aside air molecules to equal the force of gravity pulling the diver down.

Period 3 Summary

3.1: A rate is a ratio (a fraction). Commonly used rates include speed, velocity, and acceleration.

$$\frac{\text{Distance}}{\text{Time elapsed}} = \text{speed}$$ (or velocity, if the direction of motion is indicated)

3.2: A change in velocity (change in speed or direction) occurs when a net force acts on a moving object.

A force is any push or pull on an object.

The net force of all forces acting in the same direction is the sum of the forces.

The net force of forces acting in opposite directions is the difference of the forces.

3.3: Acceleration is the rate of change in velocity per change in time.

Acceleration is the result of a net force acting on an object.

For constant acceleration, $\boldsymbol{a} = \dfrac{\boldsymbol{v_f} - \boldsymbol{v_i}}{\boldsymbol{t}}$

Solving this equation for the final velocity gives $\boldsymbol{v_f} = \boldsymbol{a\,t} + \boldsymbol{v_i}$

3.4. The net force acting on an object = the product of the object's mass and its acceleration: $\boldsymbol{F} = \boldsymbol{M\,a}$

The net force on an object that is not accelerating is zero.

When a moving object is not accelerating, its velocity is constant.

An object reaches a constant velocity when the sum of the forces acting on it cancel.

3.5: The acceleration of gravity causes unsupported objects to fall toward Earth at the rate of 9.8 m/s^2 (or 32 ft/s^2 in English units).

Period 3 Exercises

E.1 Which of the following is **TRUE**?

a) 3 meters/second is not a rate.
b) 2 beers a week is not a rate.
c) 12 feet at 3 o'clock is not a rate.
d) 45 miles per hour is not a rate.
e) all of the above are rates

E.2 Which of the following statements about forces is **FALSE**?

a) A net force is required to change the velocity of an object.
b) If an object does not move, there must be no forces acting on it.
c) When two forces act in the same direction on an object, the net force equals the sum of the forces.
d) When two forces act in opposite directions on an object, the net force equals the difference between the forces.
e) **NONE** of the above statements is **FALSE**.

E.3 If you see a lightning bolt during a thunderstorm and hear the thunder from it 6 seconds later, how far away was the lighting? Assume that the speed of sound in air is about 340 meters/second and that you can see a stroke of lightning instantly.

a) 2,040 meters
b) 57 meters
c) 1.8 meters
d) 0.017 meters
e) none of the above is correct

E.4 The Mars Pathfinder spacecraft traveled about 3×10^8 miles to reach Mars. It moved at an average speed of 1.5×10^6 miles per day. How many days did it take to reach its destination?

a) 400 days
b) 365 days
c) 200 days
d) 80 days
e) 30 days

E.5 If you drive your car at 20 miles/hour and then accelerate at a rate of 3 miles/hour every second, how fast will you be going after 8 seconds?

a) 20 miles/hour
b) 24 miles/hour
c) 31 miles/hour
d) 44 miles/hour
e) 60 miles/hour

E.6 The engine of a 8,000 kg race car exerts a net force of 32,000 newtons in the horizontal direction. What is the acceleration of the car?

a) 2.9×10^{8} m/s^2
b) 4×10^{3} m/s^2
c) 4 m/s^2
d) 0.25 m/s^2
e) none of the above is correct

Period 3 Review Questions

R.1 What is the difference between speed and velocity? A device on the dashboard of your car is called a speedometer. Why isn't it called a velocity meter? How could you build a velocity meter?

R.2 Define acceleration. What caused the metal fan cart demonstrated in class to accelerate?

R.3 What can happen to the motion of an object when two forces act on it in opposite directions?

R.4 What causes falling objects to accelerate? Which falls at a faster rate – a sheet of paper slightly crumpled or a sheet crumpled into a tight ball? Why?

R.5 What can happen to the motion of a moving object when an applied force is exerted on the object? Consider the cases of applied forces that are greater than, equal to, and less than the force of friction between the object and the surface it moves across. What size of an applied force is needed to start in motion an object at rest?